Effects of matter density variations on dominant oscillations in long baseline neutrino experiments

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Abstract

Variations around the average density and composition of the Earth mantle may affect long-baseline (anti)neutrino oscillations through matter effects. For baselines not exceeding a few thousand km, such effects are known to be very small, and can be practically regarded as fractional contributions to the theoretical uncertainties. We perturbatively derive compact expressions to evaluate such contributions in phenomenologically interesting scenarios with three or four neutrinos and a dominant mass scale.

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I. INTRODUCTION

The increasing evidence in favor of neutrino flavor oscillations motivates further studies of the oscillation phenomenon by means of accelerator neutrino experiments with a (very) long baseline L. Current long baseline projects include the Japanese KEK-to-Kamioka (K2K) experiment with $L = 250$ km [1], the European CERN Neutrinos to Gran Sasso (CNGS) experiment with $L = 732$ km [2–4] (in construction), and the American Main Injector Neutrino Oscillation Search (MINOS) experiment with $L = 735$ km [5] (in construction). Possible future projects include oscillation searches with much longer baselines, e.g., using ν factories [6,7]. Although there is still a lively debate on baseline optimization for such future projects, there seems to be an increasing consensus in favor of $L \sim 2$–3×10^3 km [7,8].

For baselines $L \lesssim 10^4$ km, neutrinos cross only the Earth mantle, whose matter density can significantly affect the dynamics of flavor oscillations [9]. For baselines $L \lesssim 6 \times 10^3$ km (as considered in the present work) it has been shown extensively that the main matter effects are accurately described by approximating the mantle density profile with its average value along the ν trajectory (see [10,11] and the extensive bibliography in [12]). Therefore, variations in the matter density (and chemical composition) around the average value can be regarded as a fractional contribution to the (theoretical) error budget, rather than as a contribution to observable oscillation signals. It should be also stressed that the mantle density variations around the average value may also be affected by their own uncertainties. Indeed, the standard (radially symmetric) density profile provided by Preliminary Reference Earth Model (PREM) [13] is subject to local variations and uncertainties at the level of ~ 5–10%, which may well be nonsymmetrical along a given trajectory [14].

From the above viewpoint, one would like to have a practical recipe to evaluate the (small) density variation effects for a variety of possible density profiles (suggested, or at least permitted, by local geophysical data), without solving the ν evolution equations numerically. In this work, by using the perturbative approach applied in [15] for 2ν oscillations at first order in the density variations, we derive such a recipe for 3ν and 4ν oscillation scenarios characterized by a dominant mass scale. We think that our simple expressions may be useful to experimentalists involved in simulations of long baseline oscillation signals, in order to easily quantify the effects of uncertainties associated to (known or hypothetical) matter density variations.

II. NOTATION AND METHOD

In this section we introduce the notation and the calculation method. The kinematics is defined by the unitary matrix U connecting flavor states ν_α with massive states ν_i,

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \ , \tag{1}$$

and by the squared mass matrix

$$\mathcal{M}^2 = \mathrm{diag}(m_1^2, m_2^2, \ldots) \ . \tag{2}$$

The dynamics is the defined by the hamiltonian

$$\mathcal{H}(x) = U\,\frac{\mathcal{M}^2}{2E}\,U^\dagger + \mathcal{V}(x) \tag{3}$$

where $\mathcal{V}$ is the matrix of potentials felt by neutrinos in matter at the position x (in the flavor basis), up to irrelevant terms proportional to the unit matrix $\mathbf{1}$.

Solving the dynamics is equivalent to find the evolution operator T from the ν production point x_i to the detection point x_f,

$$\nu_\beta(x_f) = \sum_\alpha T_{\beta\alpha}(x_f,\, x_i)\ \nu_\alpha(x_i)\ , \tag{4}$$

which provides the desired flavor oscillation probabilities,

$$P(\nu_\alpha \to \nu_\beta) \equiv P_{\alpha\beta} = |T_{\beta\alpha}|^2\ . \tag{5}$$

A perturbative evaluation of T can be made [15] by splitting the ν potential matrix as

$$\mathcal{V}(x) = \overline{\mathcal{V}} + \delta\mathcal{V}(x)\ , \tag{6}$$

where

$$\overline{\mathcal{V}} = \int_{x_i}^{x_f} dx \mathcal{V}(x) \Big/ (x_f - x_i) \tag{7}$$

is the average potential matrix along the neutrino trajectory, and δV is the residual variation, characterized by

$$\int_{x_i}^{x_f} dx\ \delta\mathcal{V}(x) = 0\ . \tag{8}$$

For later purposes, it is useful to distinguish variations of the neutrino potential which are symmetric $(+)$ or antisymmetric $(-)$ with respect to the trajectory midpoint

$$\overline{x} = (x_i + x_f)/2\ , \tag{9}$$

namely,

$$\delta\mathcal{V}(x) = \delta\mathcal{V}^+(x) + \delta\mathcal{V}^-(x)\ , \tag{10}$$

where

$$\delta\mathcal{V}^\pm(x - \overline{x}) = \pm\delta\mathcal{V}^\pm(\overline{x} - x)\ . \tag{11}$$

The hamiltonian is split as

$$\mathcal{H}(x) = \overline{\mathcal{H}} + \delta\mathcal{V}(x)\ , \tag{12}$$

where the constant part is

$$\overline{\mathcal{H}} = U\frac{\mathcal{M}^2}{2E}U^\dagger + \overline{\mathcal{V}}\ . \tag{13}$$

Correspondingly, the evolution operator is split as

$$T = \overline{T} + \delta T \ , \tag{14}$$

where $\overline{T}$ is trivially obtained by exponentiating the constant part of $\mathcal{H}$,

$$\overline{T}(x_f,\, x_i) = e^{-i\overline{\mathcal{H}}(x_f - x_i)} \ , \tag{15}$$

while the correction δT (at first order in $\delta\mathcal{V}$) is perturbatively given by

$$\delta T(x_f,\, x_i) = -i \int_{x_i}^{x_f} dx\, \overline{T}(x_f,\, x)\, \delta\mathcal{V}(x)\, \overline{T}(x,\, x_i) + O(\delta\mathcal{V}^2) \ . \tag{16}$$

Finally, the oscillation probability is given by

$$P_{\alpha\beta} = |\overline{T}_{\beta\alpha} + \delta T_{\beta\alpha}|^2 \tag{17}$$

$$= \overline{P}_{\alpha\beta} + \delta P_{\alpha\beta} \ , \tag{18}$$

where

$$\overline{P}_{\alpha\beta} = |\overline{T}_{\beta\alpha}|^2 \tag{19}$$

refers to the approximation of constant neutrino potential (i.e., constant fermion density), while

$$\delta P_{\alpha\beta} = 2\,\mathrm{Re}\,(\overline{T}_{\beta\alpha}\, \delta T^*_{\beta\alpha}) + O(\delta\mathcal{V}^2) \tag{20}$$

represents the (first-order) correction due to fermion density variations along the neutrino trajectory.

In the following sections we evaluate δP for 3ν and 4ν scenarios characterized (from the point of view of long baseline experiments) by a dominant (squared) mass scale m^2, coincident with the squared mass difference indicated by current atmospheric neutrino experiments [16,17]. Our results are valid (at first perturbative order) also beyond the approximation of a dominant mass scale, provided that the subdominant oscillating terms themselves can be treated as a perturbation, which is a good approximation in many cases of phenomenological interest for present and future long baseline projects (see, e.g., [18,19]). In this case, the effect of matter density *variations* on *subdominant* oscillations can be regarded as a second order effect, which has been shown to be safely negligible unless $L \gtrsim 7 \times 10^3$ km [20,21] (a case not considered in this work).

III. RESULTS FOR THREE-NEUTRINO OSCILLATIONS

Let us consider three neutrino flavors,

$$\nu_\alpha = (\nu_e,\, \nu_\mu,\, \nu_\tau) \ , \tag{21}$$

and a mass spectrum with two practically degenerate states,

$$m_2^2 \simeq m_1^2 \ , \tag{22}$$

plus a third "lone" state which can be either heavier or lighter than the previous "doublet" of states,

$$m_3^2 - m_{1,2}^2 = \pm m^2 \ . \tag{23}$$

In the following, the double sign $\pm$ is explicitly kept to distinguish the so-called normal ($+m^2$, upper sign) and inverted ($-m^2$, lower sign) spectrum hierarchy. The mass spectrum can then be written as

$$\mathcal{M}^2 = \mathrm{diag}\left(\mp\frac{m^2}{2}, \mp\frac{m^2}{2}, \pm\frac{m^2}{2}\right) \ . \tag{24}$$

The scale m^2 and the neutrino energy E define the neutrino wavenumber relevant for the dominant oscillations,

$$k = m^2/2E \ . \tag{25}$$

The mixing matrix U is parametrized in standard form [22] as

$$U = U_{23}(\psi)U_{13}(\phi)U_{12}(\omega) \ , \tag{26}$$

where the CP phase δ is omitted, being irrelevant under the approximation in Eq. (22).

The potential matrix can be written as

$$\mathcal{V} = \mathrm{diag}\left(+\frac{V}{2}, -\frac{V}{2}, -\frac{V}{2}\right) \ , \tag{27}$$

where

$$V(x) = \sqrt{2}\, G_F\, N_e(x) \tag{28}$$

and N_e is the electron number density.[1] The above notation refers to *neutrinos*. The case of *antineutrinos* ($V \to -V$) will be discussed at the end of this section.

The constant hamiltonian $\overline{\mathcal{H}}$ is explicitly diagonalized as

$$\overline{\mathcal{H}} = U_{23}(\psi)U_{13}(\bar{\phi})\ \mathrm{diag}(\overline{E}_1, \overline{E}_2, \overline{E}_3)\ U_{13}^T(\bar{\phi})U_{23}^T(\psi) \ , \tag{29}$$

the energy eigenvalues being given by

$$\overline{E}_1 = \mp\frac{\overline{k}}{2} \ , \tag{30}$$

$$\overline{E}_2 = \frac{\mp k - \overline{V}}{2} \ , \tag{31}$$

$$\overline{E}_3 = \pm\frac{\overline{k}}{2} \ , \tag{32}$$

[1] In the Earth mantle, it is $N_e \sim 2$ mol/cm^3. See also Fig. 1 in [15].

where

$$s_{2\bar{\phi}} = \frac{s_{2\phi}}{\sqrt{(c_{2\phi} \mp \overline{V}/k)^2 + s^2_{2\phi}}} \tag{33}$$

gives the neutrino mixing angle $\bar{\phi}$ in matter ($c = \cos$, $s = \sin$), and

$$\overline{k} = k\frac{s_{2\phi}}{s_{2\bar{\phi}}} \tag{34}$$

is the neutrino wavenumber in matter (for average electron density).

By using Eqs. (15) and (19) one gets

$$\begin{aligned}\begin{pmatrix} \overline{P}_{ee} & \overline{P}_{e\mu} & \overline{P}_{e\tau} \\ \overline{P}_{\mu e} & \overline{P}_{\mu\mu} & \overline{P}_{\mu\tau} \\ \overline{P}_{\tau e} & \overline{P}_{\tau\mu} & \overline{P}_{\tau\tau} \end{pmatrix} &= \mathbf{1} + \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix} s^2_{2\psi}(c^2_{\bar{\phi}}\, S^2_{23} + s^2_{\bar{\phi}}\, S^2_{12}) \\ &\quad + \begin{pmatrix} -1 & s^2_\psi & c^2_\psi \\ s^2_\psi & -s^4_\psi & -s^2_\psi c^2_\psi \\ c^2_\psi & -s^2_\psi c^2_\psi & -c^4_\psi \end{pmatrix} s^2_{2\bar{\phi}}\, S^2_{31}\end{aligned} \tag{35}$$

for the oscillation probabilities with constant (average) potential $\overline{V}$, where the oscillation factors are given by

$$S^2_{jl} = \sin^2\left(\frac{\overline{E}_j - \overline{E}_l}{2}(x_f - x_i)\right) \ . \tag{36}$$

The above results for $\overline{P}_{\alpha\beta}$ are well-known (see, e.g., [23]).

We then compute the desired corrections $\delta P_{\alpha\beta}$ due to electron density variations through Eq. (20). Omitting the algebra, the final results read

$$\begin{aligned}\begin{pmatrix} \delta P_{ee} & \delta P_{e\mu} & \delta P_{e\tau} \\ \delta P_{\mu e} & \delta P_{\mu\mu} & \delta P_{\mu\tau} \\ \delta P_{\tau e} & \delta P_{\tau\mu} & \delta P_{\tau\tau} \end{pmatrix} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{pmatrix} s^2_{2\bar{\phi}} s^2_\psi c^2_\psi\, C\, S' \\ &\quad + \begin{pmatrix} -1 & s^2_\psi & c^2_\psi \\ s^2_\psi & -s^4_\psi & -s^2_\psi c^2_\psi \\ c^2_\psi & -s^2_\psi c^2_\psi & -c^4_\psi \end{pmatrix} s^2_{2\bar{\phi}} c_{2\bar{\phi}}\, C\, S'' \ ,\end{aligned} \tag{37}$$

where S' and S'' are oscillating terms defined as

$$S' = \sin\left(\frac{\mp k - \overline{V}}{2}(x_f - x_i)\right) \ , \tag{38}$$

$$S'' = \sin\left(\pm\frac{\overline{k}}{2}(x_f - x_i)\right) \ , \tag{39}$$

while C is basically the Fourier (cosine) transform of the symmetric part of the potential variations,

$$C(\overline{k}) = \int_{x_i}^{x_f} dx\, \delta V^+(x) \cos\left(\overline{k}(x - \overline{x})\right) \ . \tag{40}$$

By comparing Eqs. (35) and (37), it emerges that the effects of nonconstant electron density can be simply embedded as corrections to the oscillating factors S^2_{jl},

$$S^2_{jl} \longrightarrow S^2_{jl} + C \times \begin{cases} s^2_{\bar{\phi}} c^2_{\bar{\phi}}\ S' & \text{if } jl = 23 \text{ or } 12\ , \\ c_{2\bar{\phi}}\ S'' & \text{if } jl = 13\ . \end{cases} \tag{41}$$

The above replacement represent our simple recipe for the evaluation of matter density variations in the 3ν scenario. Once the usual "constant density" probabilities [Eq. (35)] have been computed for a given long baseline experiment configuration, one has just to calculate the Fourier term C [Eq. (40)] and make the substitutions in Eq. (41) to get the corrected probabilities. By using a variety of possible density profiles (allowed or suggested by geophysical information along the given baseline) one can then easily evaluate the effect of uncertainties in N_e upon observable quantities, without solving numerically the neutrino evolution equations for the specific profile.

Several remarks are in order, about the size and the properties of the correction terms $\delta P_{\alpha\beta}$ in Eq. (37). Their size is typically rather small ($\lesssim 10^{-3}$), both because they are suppressed by an overall factor $\sin^2 2\bar{\phi}$ (constrained to be $\lesssim 0.1$ by reactor[2] and atmospheric neutrino data [16]), and because the integral C is typically rather small, due to the oscillating behavior of the integrand. The maximum of C is reached only for a hypothetical configuration of the matter profile with $\delta N_e^+(x) \propto \cos[\overline{k}(x - \overline{x})]$, namely, when matter density fluctuations happen to be "in phase" with oscillations. In such a contrived case, since

$$dx\, \delta V^+ = 0.386 \times 10^{-3} \left(\frac{dx}{\text{km}}\right) \left(\frac{\delta N_e^+}{\text{mol/cm}^3}\right) \tag{42}$$

with (plausibly) $\delta N_e \lesssim 10\% N_e \simeq 0.2$ mol/cm^3, the integral C could be as large as a few percent over long baselines ($\gtrsim 10^3$ km). However, the realization of such a "maximum effect" configuration (if approximately reachable) can be attained only in a narrow range of k (i.e., of neutrino energy). Outside such a range, the size of C (and thus of $\delta P_{\alpha\beta}$) is rapidly suppressed. We refer the reader to the recent papers [24,12] for extensive numerical investigations of the size of $\delta P_{\alpha\beta}$, for various admissible $\delta N_e(x)$ profiles.

Concerning the symmetry properties of the oscillation probabilities, notice that the first-order corrections $\delta P_{\alpha\beta}$ do not involve the antisymmetric part of the potential. As a consequence, T-violation effects are absent not only at 0th order, but also at 1st order in δV,

$$\overline{P}_{\alpha\beta} = \overline{P}_{\beta\alpha}\ , \tag{43}$$

$$P_{\alpha\beta} = P_{\beta\alpha} + O(\delta V^2)\ , \tag{44}$$

in agreement with the discussion in [20,21].

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[2]Although reactor bounds apply, strictly speaking, to ϕ and not to $\bar{\phi}$, our arguments are not qualitatively changed.

Finally, let us consider the case of *antineutrinos*, for which the potential is given by $-V$. It is easy to check that all the above oscillation probabilities are invariant under the simultaneous replacements $\pm m^2 \to \mp m^2$ (change of hierarchy) *and* $+V \to -V$ (change of potential). Therefore, antineutrino probabilities for direct (inverse) hierarchy are equal to neutrino probabilities for inverse (direct) hierarchy,

$$P(\pm m^2;\, \overline{\nu}_\alpha \to \overline{\nu}_\beta) = P(\mp m^2;\, \nu_\beta \to \nu_\alpha)\ . \tag{45}$$

IV. RESULTS FOR FOUR-NEUTRINO OSCILLATIONS

In this section we consider the phenomenologically interesting scenario of three active plus one sterile neutrino ν_s, with a mass spectrum consisting of two separated doublets (see [17] and references therein). In such a case, it is useful to decompose the flavor space as

$$(\nu_e,\, \nu_\mu,\, \nu_\tau,\, \nu_s) = (\nu_+,\, \nu_\mu) \oplus (\nu_-,\, \nu_e) \tag{46}$$

where the states $\nu_\pm$ are linear combinations of ν_τ and ν_s,

$$\begin{pmatrix} \nu_+ \\ \nu_- \end{pmatrix} = \begin{pmatrix} c_\xi & s_\xi \\ -s_\xi & c_\xi \end{pmatrix} \begin{pmatrix} \nu_\tau \\ \nu_s \end{pmatrix}\ . \tag{47}$$

Under plausible approximations discussed in [17], the mass-mixing parameter space can then be factorized as

$$4\nu \text{ parameter space} \simeq (m^2,\, \psi,\, \xi) \otimes (\delta m^2,\, \omega,\, \xi)\ , \tag{48}$$

where the first and second factors dominate the oscillation physics in terrestrial and solar experiments, respectively. From the point of view of long-baseline experiments, the role of (m^2, ψ) is similar to the 3ν case in the previous section, while the new angle ξ interpolates smoothly from pure active oscillations ($s_\xi = 0$) to pure sterile oscillations ($s_\xi = 1$).

In the (ν_+, ν_μ) basis, the effective hamiltonian for long baseline experiments is given by [17]

$$\overline{\mathcal{H}} = \frac{1}{2} \begin{pmatrix} \overline{V} - k\, c_{2\psi} & k\, s_{2\psi} \\ k\, s_{2\psi} & -\overline{V} + k\, c_{2\psi} \end{pmatrix}\ , \tag{49}$$

where the average neutrino potential is now related to the average neutron density $\overline{N}_n$,

$$\overline{V} = \sqrt{2}\, G_F\, s_\xi^2\, \overline{N}_n/2\ . \tag{50}$$

The dynamics is then effectively reduced to an equivalent two-flavor problem, which is easily solved. We give the final results for the relevant muon neutrino probabilities,

$$\overline{P}_{\mu\mu} = 1 - s^2_{2\bar{\psi}}\, S^2\ , \tag{51}$$

$$\overline{P}_{\mu\tau} = c_\xi^2\, s^2_{2\bar{\psi}}\, S^2\ , \tag{52}$$

$$\overline{P}_{\mu s} = s_\xi^2\, s^2_{2\bar{\psi}}\, S^2\ , \tag{53}$$

where

$$s_{2\bar{\psi}} = \frac{s_{2\psi}}{\sqrt{(c_{2\psi} - \overline{V}/k)^2 + s_{2\psi}^2}} \tag{54}$$

and the dominant oscillating factor is

$$S^2 = \sin^2\left(\frac{\overline{k}}{2}(x_f - x_i)\right) \ , \tag{55}$$

with

$$\overline{k} = k\frac{s_{2\psi}}{s_{2\bar{\psi}}} \ . \tag{56}$$

We perturbatively find that the first-order correction due to nonconstant neutron density amounts to the following replacement

$$S^2 \to S^2 + c_{2\bar{\psi}}\, C \, \sin\left(\frac{\overline{k}}{2}(x_f - x_i)\right) \ , \tag{57}$$

where C is formally defined as in Eq. (40).

Notice that, contrary to the previous 3ν scenario, the cases $+m^2$ and $-m^2$ are now equivalent, being connected by the replacement $\psi \to \ \pi/2 - \psi$. One can then fix the sign of m^2, provided that ψ is taken in its full range $[0, \pi/2]$.

Finally, antineutrino probabilities are obtained by the above neutrino probabilities through the replacement $V \to -V$ or, equivalently, through $m^2 \to -m^2$ or, equivalently, through $s_\psi \to c_\psi$.

V. SUMMARY

In the context of long baseline ν oscillation experiments, the small effects due to nonconstant matter density (along a given neutrino trajectory) can be regarded as a fractional contribution to the uncertanties associated to oscillation signals. Therefore, it makes sense to search for a practical method to compute them for given density profiles. We perturbatively find that the first-order effects can be simply embedded as a correction to the oscillating factors in both 3ν and 4ν scenarios with a dominant mass scale [Eqs. (41) and (57), respectively]. The correction involves the Fourier (cosine) transform of the symmetric part of the density variations, which can be easily evaluated and incorporated in experimental simulations.

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